CONTROL AND OPTIMIZATION OF COHERENCE OF A NANO-SIZED SPIN-TORQUE MICROWAVE OSCILLATOR FOR MILITARY NANO-ELECTRONICS

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ABSTRACT

A theory of generation linewidth of a spin-torque oscillator (STO) based on an in-plane magnetized nano-pillar with an anisotropic “free” magnetic layer has been developed. It is shown that the coherent properties of microwave generation strongly depend on the direction and magnitude of the in-plane bias magnetic field. In particular, it is predicted that by choosing the direction of the bias field H_0 along the “hard” anisotropy axis of the STO “free” layer and the magnitude of this field to be four times larger than the anisotropy field H_A ($H_0 = 4H_A$) it would be possible to compensate the nonlinear phase noise and to achieve the minimum value of the generation linewidth, characteristic for an auto-oscillator *without* a nonlinear frequency shift. The developed theory of STO creates a possibility to unite nano-magnetism and microwave theory, and to develop a basis for a novel nano-spintronic microwave technology – the technology of nano-sized tunable microwave oscillators that are insensitive to ionizing radiation and, therefore, are suitable for applications in the future military nano-electronic integrated circuits.

1. INTRODUCTION

Recent discovery of the *spin-torque* effect (Slonczewski, 1996; Berger, 1996; Tsoi *et al.*, 1998; Kiselev *et al.*, 2003) in magnetic multilayers driven by direct electric current opens a possibility for the development of a novel type of microwave auto-oscillator – a *spin-torque oscillator* (STO), which consists of a nano-scale metallic contact attached to a magnetic multilayer or a multilayered magnetic nano-pillar (Slavin and Tiberkevich, 2008). The spin-torque effect is an effect that is opposite to the effect of *giant magnetoresistance*, for which the 2007 Nobel Prize was awarded.

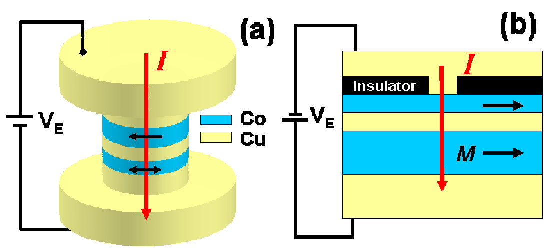

Fig. 1. Geometries of nanoscale spin-torque oscillators (STO) based: **(a)** on a magnetic nano-pillar where the lateral size of the magnetic layer is fixed (diameter ~ 100 nm)**; (b)** on a magnetic nano-contact where the size of the contact region in fixed (diameter ~ 40 nm).

Typical geometries of STO based on magnetic nano-pillars and nano-contacts are presented in Fig.1

The STOs are nano-sized, compatible with the existing planar technology, and the frequency generated by these auto-oscillators is determined by the applied magnetic field, static magnetization, *etc.*, and is, in general, close to the ferromagnetic resonance (FMR) frequency ω_0 of the “free” magnetic layer (upper blue layer in Fig.1).

From the practical point of view, one of the most important characteristics of any auto-oscillator is the degree of coherence of the generated oscillations. The convenient measure of the coherence of auto-oscillations is the generation linewidth, which is proportional to the inverse of the decoherence time.

Due to small nano-scale sizes of STO and, respectively, rather small energy of oscillations (which can be comparable to the thermal energy $k_B T$ at room temperature) thermal fluctuations play an important role in the dynamics of spin-torque oscillations. As a result, generation linewidth of nano-sized STO is much larger than the typical linewidths in conventional macro-sized

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microwave auto-oscillators and the problem of control and optimization of the coherent properties of STO becomes very important.

Like in any conventional auto-oscillator the generation linewidth of an STO is determined by the phase noise caused by thermal fluctuations, but in contrast with the conventional case, the generation frequency $\omega(p)$ of an STO is strongly dependent on the oscillation power p, i.e. a nonlinear shift of generation frequency takes place. This nonlinear frequency shift allows one to tune the generation frequency of the STO, but, at the same time, creates an additional source of the phase noise, which leads to a significant broadening of the generation linewidth (Sankey *et al*., 2005; Kim, Tiberkevich, and Slavin, 2008). This nonlinear renormalization of the phase noise is the dominant mechanism of decoherence and linewidth broadening in the case of STO.

Since the nonlinear frequency shift of an STO changes its sign with the variation of the out-of-plane magnetization angle (see e.g. (Slavin and Tiberkevich, 2008) for details) it is possible to choose the *out-of-plane* magnetization angle, at which the nonlinear frequency shift is close to zero. Both the experiment (Rippard, Pufall, and Russek, 2006) and theory (Kim, Tiberkevich, and Slavin, 2008) confirmed that the minimum generation linewidth of an STO, based on an *isotropic* “free” layer, magnetized at an angle θ_0 to the “free” layer plane, is achieved for the magnetization angle $\theta_0 \approx 80°$, at which the generation frequency is almost independent of power, $\omega(p) \approx \mathrm{const}$. At this magnetization angle the STO behaves as a conventional “linear” oscillator without a nonlinear frequency shift.

Operation of STO with almost normally magnetized “free” layer ($\theta_0 \approx 80°$), however, requires use of large bias magnetic fields, compared with the saturation magnetization of the “free” layer, which is undesirable in most practical applications. From the practical point of view, it would be much better to use *in-plane* magnetized “free” layer.

In this paper, we develop a theory of generation linewidth of an STO based on a magnetic nano-pillar with an *anisotropic in-plane magnetized* “free” magnetic layer. We show, that even a relatively small in-plane anisotropy strongly modifies the nonlinear properties of the magnetization precession, and leads to a significant dependence of the STO linewidth $\Delta\omega$ on the angle ϕ_0 that the in-plane bias magnetic field H_0 makes with the easy axis of anisotropy of the “free” layer. We, also, demonstrate that by a proper choice of the magnitude and direction of the in-plane bias magnetic field it is possible to minimize the generation linewidth of an anisotropic STO, similar to the case of STO with the out-of-plane magnetization (Rippard, Pufall, and Russek, 2006), and to significantly increase the coherence time of the generated microwave auto-oscillations.

2. THEORY

The problem of generation linewidth of an arbitrary nonlinear auto-oscillator has been considered in (Kim, Tiberkevich, and Slavin, 2008). It has been shown, that for sufficiently small temperature the generation linewidth of an oscillator can be written as

$$\Delta\omega = \Gamma_+(p_0)\frac{k_B T}{E(p_0)}\left[1+\left(\frac{N}{\Gamma_{\mathrm{eff}}}\right)^2\right], \tag{1}$$

where, in case of STO, $\Gamma_+(p) = \Gamma_G(1+Qp)$ is the natural positive damping of an oscillator, which depends on the dimensionless oscillation power p, Γ_G is the linear Gilbert damping rate, Q is the phenomenological nonlinear damping parameter that account both for the intrinsic nonlinear damping mechanisms and for the nonlinear energy transfer between different modes of an oscillator. p_0 is the average power of auto-oscillations, k_B is the Boltzmann constant, T is the absolute temperature, $E(p) = \beta p$ is the energy of oscillations, $\beta = (M_0/\gamma)\omega_0 V_{\mathrm{eff}}$ is the power-energy proportionality coefficient, M_0 is the saturation magnetization of the “free” layer, γ is the gyromagnetic ratio, and V_{eff} is the effective volume of the magnetic material of the “free” layer involved in auto-oscillations.

The second term inside the square brackets in Eq. (1) describes the linewidth broadening due to the dependence of the oscillator frequency $\omega(p)$ on the power p. Here $N = d\omega(p)/dp$ is the nonlinear frequency shift parameter, and the effective damping Γ_{eff} is defined as

$$\Gamma_{\mathrm{eff}} = \frac{d\Gamma_+(p)}{dp} - \frac{d\Gamma_-(p)}{dp} = \Gamma_G(\zeta + Q), \tag{2}$$

where $\Gamma_-(p) = \sigma I(1-p)$ is the effective negative damping created by the bias current I, coefficient σ is defined by Eq. (2) in (Slavin and Tiberkevich, 2006), $\zeta = I/I_{\mathrm{th}}$ is the supercriticality parameter, and $I_{\mathrm{th}} = \Gamma_G/\sigma$ is the threshold current, at which the self-sustained oscillations in the STO start to appear.

All the derivatives in (2) are taken at the stationary auto-oscillation power $p = p_0$, which is determined from the condition of the energy balance $\Gamma_+(p_0) = \Gamma_-(p_0)$:

$$p_0 = \frac{\zeta - 1}{\zeta + Q}. \tag{3}$$

To find the dependence of the Gilbert damping rate Γ_G and the nonlinear frequency shift coefficient N on the STO parameters, we consider an anisotropic "free" layer with an in-plane anisotropy field H_A, magnetized by the in-plane bias magnetic field $H_0 > H_A$ directed at the angle ϕ_0 with respect to the easy axis of the "free" layer (see inset in Fig. 2). The effective anisotropy field H_A accounts both for the crystallographic anisotropy of the "free" layer and for the shape anisotropy caused, in the case of an STO based on magnetic nano-pillar, by the non-circular shape of the pillar. Similarly, the bias field H_0 is a sum of the applied magnetic field and the dipolar field created by the "fixed" layer of the STO. We assumed that current excites a spatially-uniform spin wave mode of a magnetic nano-pillar.

In our derivation we used classical Hamiltonian formalism for spin waves and performed the renormalization of the non-resonant three-wave nonlinear processes (see (Slavin and Tiberkevich, 2008) for details), which significantly modified the dependence of the nonlinear coefficient N on the in-plane magnetization angle ϕ_0.

The obtained analytical expressions for Γ_G and N have the form:

$$\Gamma_G = \alpha_G A, \tag{4a}$$

$$N = -\frac{1}{\omega_0 A}\Big[\omega_0^2(2 - 3B/\omega_\parallel)B + \omega_A(2A^2 + B^2)\cos 2\phi \\ \frac{3\omega_A^2}{4\omega_0^2}\omega_\parallel(\omega_\parallel^2 + A^2)\sin^2 2\phi\Big], \tag{4b}$$

where α_G is the Gilbert damping parameter,

$$A = \omega_H + \frac{\omega_M - \omega_A \sin^2\phi}{2}, \tag{5a}$$

$$B = \frac{\omega_M + \omega_A \sin^2\phi}{2}, \tag{5b}$$

$\omega_\parallel = \omega_H + \omega_M$, ω_0 is the FMR frequency:

$$\omega_0^2 = (\omega_H - \omega_A \sin^2\phi)(\omega_H + \omega_M), \tag{6}$$

$\omega_H = \gamma H$, $\omega_M = 4\pi\gamma M_0$, $\omega_A = \gamma H_A$. The H and ϕ are the magnitude and in-plane angle of the *internal* magnetic field, which are connected with the *external* values H_0 and ϕ_0 by the expressions

$$(H - H_A)\cos\phi = H_0 \cos\phi_0, \tag{7a}$$

$$H \sin\phi = H_0 \sin\phi_0. \tag{7b}$$

For relatively small bias fields ($H_0, H_A << 4\pi M_0$) one can simplify the above expressions in the cases when the external bias magnetic field is directed along either easy ($\phi_0 = 0$) or hard ($\phi_0 = \pi/2$) in-plane anisotropy axes:

$$\omega_0 \approx \gamma\sqrt{4\pi M_0 (H_0 \pm H_A)}, \tag{8a}$$

$$\Gamma_G \approx \alpha_G\, \omega_M/2, \tag{8b}$$

$$N \approx -\gamma(H_0 \pm 4H_A)\omega_M/(2\omega_0). \tag{8c}$$

Here the upper (lower) signs correspond to the magnetization direction along the easy (hard) axis.

Derived Eqs. (4) – (7) together with the general expression (1) for the generation linewidth of a nonlinear auto-oscillator completely solve the problem of the linewidth of an anisotropic in-plane magnetized spin-torque auto-oscillator.

3. ANALYSIS AND DISCUSSION

Fig. 2 shows the dependence (4b) of the nonlinear frequency shift coefficient N on the magnetization angle ϕ_0, whereas the angular dependence (1) of the STO linewidth $\Delta\omega$ is shown in the main panel of Fig. 3. One can see from Fig. 2 that N changes substantially as the bias field rotates from the easy ($\phi_0 = 0$) to hard ($\phi_0 = \pi/2$) axis orientation. At the same time, the other parameters of the STO, such as FMR frequency ω_0 and threshold current I_{th} (see inset in Fig. 3), experience much weaker angular variations. Therefore, as it follows from Eq. (1), the angular dependence of the linewidth $\Delta\omega$ almost exactly follows the dependence of the squared nonlinear frequency shift N^2 on the angle ϕ_0.

In particular, for relatively weak bias fields $H_A < H_0 < 4H_A$ the nonlinear frequency shift N changes its sign at a certain intermediate magnetization

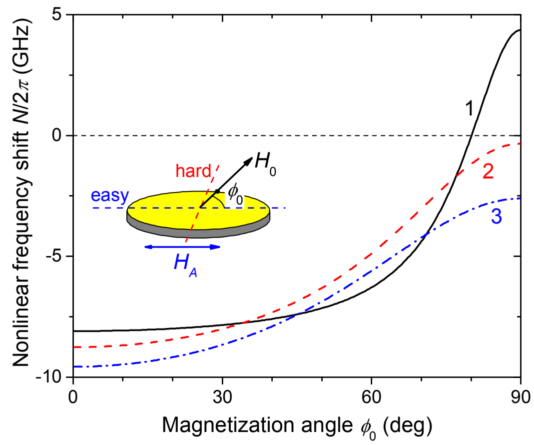

Fig. 2. Main panel: Nonlinear frequency shift coefficient N for an *anisotropic* magnetic film (saturation magnetization $4\pi M_0 = 8\,\text{kG}$, anisotropy field $H_A = 0.3\,\text{kOe}$) as a function of the angle ϕ_0 between the *in-plane* bias magnetic field and the easy axis of the film for several magnitudes of the bias magnetic field H_0: 1 – $H_0 = 2H_A = 0.6\,\text{kOe}$, 2 – $H_0 = 4H_A = 1.2\,\text{kOe}$, 3 – $H_0 = 6H_A = 1.8\,\text{kOe}$. Inset: Schematic view of the "free" layer of an STO showing the easy and hard magnetization axes and the direction of the bias field H_0.

angle ϕ_* (see curve 1 at Fig. 2, $\phi_* \approx 80°$). At this angle $N^2 = 0$ and the STO does not experience a nonlinear broadening of the generation linewidth (see Eq. (1)), and $\Delta\omega$ has a well-pronounced minimum (see curve 1 at Fig. 3).

For larger magnetic fields $H_0 > 4H_A$ the absolute value of the nonlinear frequency shift $|N|$ monotonically decreases as the field orientation rotates from the easy to hard axis and, consequently, $\Delta\omega$ has a minimum for the hard-axis magnetization $\phi_0 = \pi/2$ (see curve 3 in Figs. 2 and 3). However, since in this case $|N| \neq 0$, the linewidth minimum is not so deep as for the smaller magnetic fields (compare curves 1 and 3 at Fig. 3).

For the bias magnetic field $H_0 \approx 4H_A$ the nonlinear frequency shift N vanishes exactly at the hard axis magnetization $\phi_0 = \pi/2$ (see Eq. (8c), and curve 2 at Fig. 2) and, therefore, the linewidth $\Delta\omega$ at this point has the minimum possible value corresponding to the linewidth of a "linear" oscillator (see curve 2 at Fig. 3).

Thus, our theory predicts that to achieve the minimum generation linewidth in an STO based on an anisotropic nano-pillar it is necessary to direct the in-plane bias magnetic field along the "hard" anisotropy axis of the "free" layer and to choose the magnitude of this field close to $H_0 = 4H_A$.

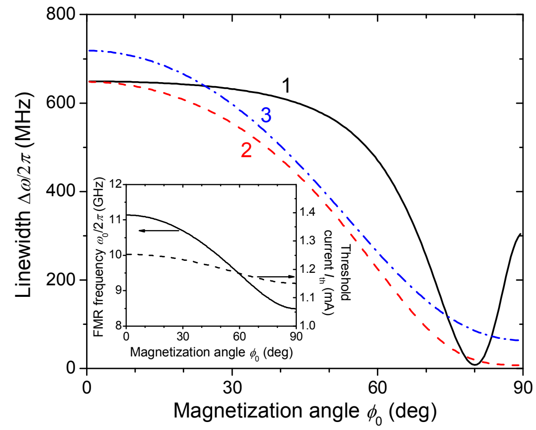

Fig. 3. Main panel: Dependence (1) of the generation linewidth $\Delta\omega$ of an STO on the in-plane magnetization angle ϕ_0 for several values of the bias magnetic field H_0: 1 – $H_0 = 2H_A = 0.6\,\text{kOe}$, 2 – $H_0 = 4H_A = 1.2\,\text{kOe}$, 3 – $H_0 = 6H_A = 1.8\,\text{kOe}$. The other parameters of the STO were chosen as: $4\pi M_0 = 8\,\text{kOe}$, $H_A = 0.3\,\text{kOe}$, $\alpha_G = 0.01$, $Q = 3$, "free" layer thickness $d = 3\,\text{nm}$, shape of the nano-pillar – circular with the radius $R = 50\,\text{nm}$, spin-polarization efficiency $\varepsilon = 0.2$, bias current $I = 3\,\text{mA}$, temperature $T = 300\,\text{K}$. Inset: Dependence of the FMR frequency ω_0 (left axis) and threshold current I_{th} (right axis) on the magnetization angle ϕ_0 for $H_0 = 4H_A = 1.2\,\text{kOe}$. Other parameters of the STO are the same as in the main panel.

4. COMPARISON WITH EXPERIMENTS

Measurements of the angular dependence of the generation linewidth of an in-plane magnetized anisotropic STO have been performed in recent experiment (Thadani *et al.*, 2008). The experimentally studied STOs had "free" magnetic layer made of permalloy (Py) and nano-patterned to have elliptical in-plane shapes with aspect ratio from 1:2 to 1:3. Since permalloy magnetic films have vanishing crystallographic anisotropy, the in-plane anisotropy of the "free" layers had magnetostatic origin, i.e. was caused only by non-circular shapes of the nano-pillars and was relatively small – about 0.1 – 0.2 kOe with the "easy" magnetization axis directed along the large axis of the nano-pillar. In spite of relatively small values of the in-plane anisotropy

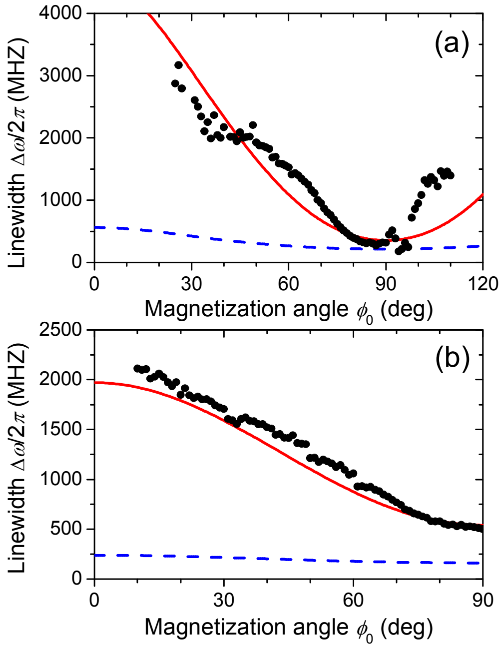

Fig. 4. Dependence (1) of the generation linewidth $\Delta\omega$ on the in-plane magnetization angle ϕ_0 for two STOs based on anisotropic magnetic nano-pillars: (a) STO with a synthetic "fixed" layer Py-IrMn; (b) STO with a thick Py "fixed" layer (after (Thadani *et al.*, 2008)). Dots – experimental data from (Thadani *et al.*, 2008), solid lines – calculation using Eq. (1) for a "nonlinear" oscillator, dashed lines – calculation using the classical "linear" expression (Eq. (1) with $N = 0$) multiplied by 10. All the calculations were done for the parameters of the experiment (Thadani *et al.*, 2008): saturation magnetization $4\pi M_0 = 8$ kOe, bias magnetic field $H_0 = 1.08$ kOe (a) and $H_0 = 1.2$ kOe (b), in-plane anisotropy field $H_A = 0.2$ kOe (a) and $H_A = 0.1$ kOe (b), Gilbert damping constant $\alpha_G = 0.015$, nonlinear damping coefficient $Q = 3$, "free" layer thickness $d = 4$ nm, shape of the nano-pillar – elliptical with the sizes 150 nm x 50 nm (a) and 130 nm x 70 nm (b), spin-polarization efficiency $\varepsilon = 0.32$ (a) and $\varepsilon = 0.375$ (b), current $I = 5$ mA, temperature $T = 300$ K.

field, the authors of (Thadani *et al.*, 2008) observed a substantial narrowing of the generation linewidth as the in-plane bias magnetic field rotated from the "easy" to "hard"-axis orientation.

These experimental findings can be easily explained by the theory developed in the present paper. In Fig. 4 we showed comparison of our theoretical results (solid lines) with the experimental data (dots) for two STOs experimentally studied in (Thadani *et al.*, 2008): (a) exchange-biased STO (see Fig. 2c in (Thadani *et al.*, 2008)) and (b) thick-fixed-layer STO (see Fig. 2d in (Thadani *et al.*, 2008)). In our theoretical calculations we used the spin-polarization efficiency ε as the only fitting parameter, whereas all the other parameters of the model were taken from the experiment or fixed at typical values for Py magnetic films (see caption in Fig. 4). We would like to stress, that the angular variations of the linewidth $\Delta\omega$ are almost independent of the value of ε since they are determined mostly by the angular dependence of the nonlinear frequency shift coefficient N.

One can see from Fig. 4 that our nonlinear single-mode analytic theory for the linewidth Eq. (1) gives a description of the experimental results (Thadani *et al.*, 2008) that is as good as the one provided by the micromagnetic simulations done in (Thadani *et al.*, 2008). The reason for that is that in our analytic model we include nonlinear dissipation (described by the parameter Q in Eqs. (2) and (3)), which implicitly accounts for the energy transfer from the main (quasi-uniform) spin wave mode excited by spin-polarized current to spatially-nonuniform spin wave modes. On the other hand, standard single-mode macrospin simulations, performed in (Thadani *et al.*, 2008), could not reproduce the experimental results since, without account of nonlinear inter-mode energy exchange, microwave magnetization precession is found to be stable in much narrower interval of parameters.

It is clear from Fig. 4 that the above presented nonlinear theory of generation linewidth of anisotropic STO satisfactory describes both the quantitative values of the linewidth and qualitative dependence of the linewidth on the in-plane magnetization angle ϕ_0. At the same time, the classical theory of the generation linewidth of a "linear" oscillator, that does not take into account the nonlinear frequency shift N (Eq. (1) with $N = 0$, see dashed lines in Fig. 4), predicts a much smaller values of the generation linewidth $\Delta\omega$ (note, that in Fig. 4 data for a "linear" oscillator have been multiplied by 10) and a much weaker dependence of the linewidth on the magnetization angle ϕ_0.

It is interesting to note, that in both cases studied in (Thadani *et al.*, 2008) and shown in Fig. 4 the bias magnetic field H_0 was larger than the theoretically predicted optimal value, $H_0 > 4H_A$ (this case corresponds to curve 3 in Figs. 2 and 3), and, in full agreement with the theory, the minimum linewidths are observed exactly at the "hard"-axis direction of magnetization. However, these minima of the generation minimum are much larger than the theoretical ("linear") minimum corresponding to the magnitude of the "hard"-axis bias field equal to $H_0 = 4H_A$, at which the coefficient of the nonlinear frequency shift N vanishes. As it follows from the theoretical calculations, it is possible to further reduce the generation linewidth by more than 10 times by deliberately choosing the "hard"-axis bias field to be equal to $H_0 = 4H_A$. It would be very interesting to check experimentally this theoretically predicted effect in a series of systematic experiments performed in an in-plane magnetized anisotropic STO.

5. CONCLUSIONS

In conclusion, we developed a theory of generation linewidth of a spin-torque oscillator with an *in-plane magnetized anisotropic* "free" layer. The developed theory predicts that the coherent properties of microwave generation in an STO can be easily controlled by the direction and magnitude of the in-plane bias magnetic field. In particular, by choosing the bias field to be oriented along the "hard"-axis direction and its magnitude to be equal to $H_0 = 4H_A$ it is possible to completely compensate nonlinear phase noise in an STO and significantly increase coherence time of microwave generation. The theory gives a reasonably good qualitative and quantitative description of recent experiments.

6. RELEVANCE TO THE ARMY GOALS

Military applications constantly require miniaturization of electronic elements, reduction of power consumption of these elements, and ability of these elements to successfully perform under the influence of ionizing radiation. Currently used generators of microwave signals are based on p-n junctions in semiconductors and, therefore, cannot withstand ionizing radiation. The above described STO represent an entirely new type of microwave oscillators – oscillators based on spin-polarized current passing through a nano-structure made of ferromagnetic metallic multi-layers. Such structures are nano-sized, could be produced using a well-established planar technology, and, being metallic, are naturally indifferent to the ionizing radiation. Therefore, STO can become active microwave elements of choice for the military nano-electronics of the near future.

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